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AN ESSAY ON SIR ISAAC NEWTON's Second LAW of MOTION.

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THE following Eſſay was preſented to the Royal Society, but not printed in their Tranſactions. It is hoped that the publiſhing it, as an addition to the former eſſays of the author, will not be thought improper by any of that Body. Nullius in verba *is the motto and maxim of the Society; and this the author alſo hopes may be his apology for queſtioning the Logic of ſo great a maſter as* NEWTON.

BRITISH MUSEUM

ESSAY

ON

Sir ISAAC NEWTON'S

SECOND LAW OF MOTION.

THE deſign of Newton in the *Principia*, is to refer all that variety of motions we ſee in the univerſe to a few general principles. All philoſophy conſiſts in tracing out general principles by way of Analyſis; and from general principles thus eſtabliſhed, deriving Synthetically a vaſt variety of particular Phaenomena*. Such general principles in *rational mechanics*,† are called rules or LAWS OF MOTION; they are circumſtances which we ſee take place in the motion of all bodies we are conversſant converſant with, and are analogous to Axioms in abſtract reaſoning. Axioms are truths which the mind aſſents to, as ſoon as clearly underſtood; they cannot be *proved*, but are the baſis of all demonſtration. The enquiry into the relation which the various motions in the univerſe have to each other, not being an enquiry into the relation of abſtract ideas; the firſt principles here, are not ſelf-evident truths, but Laws founded on univerſal experience. As in abſtract ſcience ſo here; the fewer firſt principles are aſſumed by way of Axioms, the more ſcientific is the Syſtem of Philoſophy built upon them *. Again, the Axioms of Euclid contain in them *virtually* all his Geometrical propoſitions; for theſe propoſitions are only ſo many conſequences derived from the Axioms: So the laws of motion contain in them, virtually, a determination of the motion of all bodies in this our

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* Optics, at the end.

† Rational or theoretical, as oppoſed to practical. See preface to the Principia.

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* It is not eaſy to determine what Newton meant by his *Æthereal Medium*, (Optics, Quere 19.) but it is certain he intended thereby to Generalize further than he had done in his Principia. He thought he had diſcovered a grand univerſal principle, and that from ONE ſimple Law he could derive, not only thoſe kind of motions in bodies, we diſtinguiſh from others by imputing them to Gravity; but alſo a vaſt variety of motions of another ſort; diſtinguiſhed, by being imputed to Magnetiſm, Electricity, &c. as their cauſe.

Syſtem, remote as well as near; for the particular circumſtances of theſe motions may be derived from, and are neceſſary conſequences of thoſe general Laws[*].

But to ſhew that the motion of all bodies in this our Syſtem are conſequences of the general laws laid down as axioms, we muſt compare theſe motions with reſpect to their quantities, their directions, &c. and ſhow that they have exactly (not in a looſe way nearly) that very proportion to each other which thoſe laws demand. Now whatever is thus made the ſubject of mathematical compariſon[†], muſt have ſome mathematical meaſure of its quantity.

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[*] And here we may obſerve the proper buſineſs of natural philoſophy is not, in a ſtrict ſenſe, to aſſign the cauſes of motion, but to generalize and reduce all ſorts of motion to as few claſſes as poſſible; thus to demonſtrate the wiſdom of HIM who impoſed theſe Laws upon matter, and has made all things in number, weight, and meaſure. See Principia, Def. VIII.

When we ſay that the motion of any bodies is cauſed by Gravity, we mean only that their motion is ſuch as according to theſe Laws would be the conſequence of a force varying in a *reciprocal duplicate ratio of their diſtance from the central body*. If the motion of any bodies be of a different ſort, we diſtinguiſh it by imputing their motion to ſome other unknown cauſe; ſuch as what we call Magnetiſm, Electricity, &c. Thus effects which we do ſee and underſtand, are claſſed and diſtinguiſhed, by the name we give to their ſuppoſed cauſe, of which we know nothing.

[†] Newton calls his book, *The* Mathematical *principles of Natural Philoſophy*.

It muſt be obſerved here, that ſome quantities have a meaſure in themſelves, and are therefore capable of mathematical compariſon; while others which admit of more or leſs, and that in all poſſible degrees, yet having no ſuch meaſure, cannot be *mathematically* compared. Thus numbers have a meaſure in themſelves, are made up of parts which bear a proportion to themſelves and to the whole; we can add one number to another equal number, and every one ſees, the whole is juſt double of each part. Pleaſure and pain, heat and cold, alſo admit of various degrees, but who can meaſure them?* Who can add one degree of heat to another equal degree of heat? Now to make quantities which have not a meaſure in themſelves the ſubject of mathematical compariſon, an arbitrary meaſure is aſſigned to them; by referring them to ſome quantity which has a meaſure, and to which they are related. Thus he who graduates a Thermometer, eſtabliſhes an arbitrary meaſure of heat and cold. He refers the degrees of heat to extenſion, to which it has a relation, becauſe heat increaſes the extenſion of moſt bodies; for he eſtimates the degrees of heat by the *length* of that part of the tube occupied by the Quickſilver, as it is meaſured on his ſcale. And thus we may ſay that the heat of a moderate ſummer's day, when the Thermometer ſtands at 64, is juſt double the heat at the beginning of a Froſt, when the Thermometer ſtands at 32. *

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* There are moraliſts, who not only compare *mathematically* the merit or demerit of one man with that of another, but alſo demerit with puniſhment.

In the ſame manner velocity or ſwiftneſs has no meaſure in itſelf. We perceive that ſome motions are ſwifter than others, but how ſhall we know when one motion is juſt twice or three times as ſwift as another? Now the ſpace deſcribed in a given time depending on the degree of velocity, is with great propriety made the meaſure of velocity.

So alſo *Force* is not meaſurable in its own ſelf. One force may undoubtedly be greater or leſs than another; and that whether its action is continued for ſome time, and is what we call preſſure; or whether its action be inſtantaneous or what is called impact. But who can determine when one ſpring puſhes juſt as ſtrong again as another? or one man ſtrikes juſt as hard again as another? We can no more add one blow to another equal blow, than we can add the whiteneſs of one piece of paper, to that of another juſt as white, and ſay it makes a colour as white again. Even the equality of many quantities, which have no meaſure in themſelves; (ſuch as pleaſure, pain, and all moral qualities) can hardly be aſcertained.

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* See this ſubject fully diſcuſſed in the Philoſophical Tranſactions, No. 489.

Newton perfectly aware of this, begins the Principia with eſtabliſhing *meaſures* of all thoſe quantities he intends to make the ſubject of Mathematical compariſon, and which have not a meaſure in themſelves. Among theſe are Quantity of Matter, *Def.* 1. Quantity of Motion, *Def.* 2. &c. all of which are referred for their meaſure, ultimately, to ſuch quantities as have naturally a meaſure in themſelves.

As motion is the conſequence of a force acting on any body, ſo it is with great propriety made the meaſure of that force. Motion in itſelf indeed, has no natural meaſure; but an artificial one is aſſigned to it in the ſecond definition.

By Newton's Eighth definition, the quantity of the *vis centripeta motrix*, or ſuch a force as acting continually generates motion in any body, is to be eſtimated, (that is its proportion to any other force of the like kind determined) by the quantity of motion generated by the action of that force, uniformly continued for a given time; or as it is uſually expreſſed, the quantity of force is to be meaſured, by the quantity of motion. As far as appears, the motion generated inſtantaneouſly, is alſo to be the meaſure of an inſtantaneous force. If therefore a body be already in motion, and that motion be increaſed or diminiſhed by the action of ſome new force (of either kind) ſurely that adventitious force OUGHT to be meaſured in the ſame manner as the original force was. For would it not introduce the utmoſt confuſion, to meaſure the original force by one rule, and the additional part by another? And if we lay it down as a *definition*, that the change of motion is the proper meaſure of the force ſo impreſſed, is it not identical to lay it down alſo as a *propoſition* (in the ſecond law of motion) that the change of motion is proportional to the moving force impreſſed? And what is the proof? namely, " that if any force generate a particular degree of motion, a double force will generate a double motion; whether in the way of preſſure or of impact." This ſurely, if any thing can be, is identical.

What has been ſaid may be applied to the other part of the Second Law of motion; " that the change of motion is produced in the direction of the right line in which this force is impreſſed. For there is no other mark to know in what direction any force is impreſſed, but the direction of the motion produced in conſequence of that force. As the quantity of motion produced, is made the *meaſure* of the *quantity* of force; ſo the direction of the motion is ever the *mark* of the *quality*, or direction of the force.

Newton certainly intended this law of motion ſhould be a more general propoſition, than what he derives from it in his firſt corollary (*accedet igitur corpus*, &c.) Whatever that propoſition was, he ſeems to have thought that it followed *a priori* from the nature of motion. For he does not eſtabliſh this law, as he does the other two by appealing to facts; but by arguments drawn from the nature of motion. Whether this attempt to go higher, and generalize more, has not introduced obſcurity into the law itſelf, and embarraſſed the argument by which he endeavours to prove the truth of the firſt corollary, (all that is material to be eſtabliſhed) let others judge. What follows is an attempt to expreſs this law, in more intelligible (though perhaps leſs general) terms, and to put its proof, like that of the other laws, upon univerſal experience.

DEFINITION. By the *determination* of a force is meant either the line of its direction, or any other line parallel to it*.

LAW. When a body is acted upon at once by two forces, its quantity of motion eſtimated in the reſpective determinations of thoſe forces; is the ſame when they act jointly, as when they act ſeparately.

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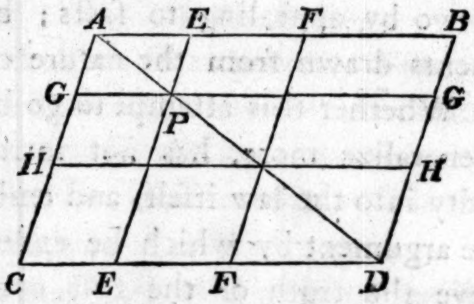
* This definition is borrowed from Saunderſon's introduction to fluxions.



To explain this, deſcribe the parallelogram *ABCD*, and draw the lines *EE*, *FF*, parallel to *AC*; alſo *GG*, *HH*, parallel to *AB*.

Let a body *A*, be acted upon in the direction *AB* by a force that would carry it in a given time, through the line *AB*. Again, let the ſame body be acted upon in the direction *AC*, by another force, that would carry it in the given time through the line *AC*. Let now theſe two forces act at the ſame time jointly on the body; then it will be carried juſt as far by their joint action in the determinations *GG*, *HH*, *CD*, as it would be by the former force alone, in the ſame determination, or in the line *AB*. It will alſo be carried juſt as far in the determinations *EE*, *FF*, *BD*, as it would by the latter force alone in its determination, or in the line *AC*. The action of the former force, though mixed with the action of the latter, will yet have no effect upon the body, but in its proper determination *GG*, *HH*, *CD*; nor will its effect, *thus eſtimated*, be increaſed, or diminiſhed, or the body be made to approach to, or recede from the line *BD* by the acceſſion of the latter force. In like manner neither will the action of the latter force (ſo eſtimated) be changed by being joined with the former. Its effect upon the body in its own proper determination *EE*, *FF*, *BD* will not be altered; nor will the body by the joint action of the former force be made to approach to or recede from the line *CD* parallel to *AB*. Thus we may ſay, neither the quantity nor the quality of the motion generated, will be at all altered by the joint action of two forces upon any body: it will be the ſame (ſo eſtimated) as if they had acted ſeparately.

What has been ſaid is only to illuſtrate the meaning, not to prove the truth of this law.

SCHOLIUM. The rule which thoſe motions follow, that are occaſioned by the action of a ſingle force, are laid down in the firſt law of motion: In this ſecond law, we have the rule which motions ariſing from the combined action of two forces (of either kind) always follow *. Newton explains (in his firſt corollary) thoſe circumſtances which always attend theſe motions, as we have done, (*nihil mutabit velocitatem*, &c.); but then ſurely theſe circumſtances ſhould have been the law itſelf, not a conſequence from it, implied in the words *Nam quoniam vis*, &c.——This law reſpects the motion of a body ſo acted upon, as it is eſtimated in the determination of each force ſeparately; but from thence the line which the body will deſcribe, and its motion eſtimated in its own proper path, may be derived as follows.

COR. I. At the end of the time in which the body by the former force would have deſcribed the ſide *AB*, or by the latter force the ſide *AC*; the body when acted upon by the two forces conjointly will be found in *D*, the oppoſite angle of the parallelogram. For by this law the former force does not change the approach of the body to the line *CD*, therefore at the end of the time it will be found ſomewhere in that line. By the ſame law, the latter force does in no wiſe change the approach of the body to the line *BD*, therefore at the end of the time it will be found ſomewhere in that line alſo; and therefore be found in their concourſe *D* the oppoſite angle of the parallelogram.

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* The third law, reſpects the communication of motion from one body to another; whether inſtantaneous, or gradual.

SCHOLIUM. As this law obtains in the joint action of two forces, of either kinds (inſtantaneous or continual) the body will in either caſe be found in the point *D*; but the road by which it gets to *D*, and its rate of travelling, will be different according to the nature of the forces combined together.

COR. II. Draw *AD* the diagonal of the parallelogram, and let the two forces act inſtantaneouſly. I ſay in this caſe the body *A*, will deſcribe the Diagonal *AD* with an uniform motion, and in the ſame time in which it would have deſcribed the ſides by theſe reſpective forces ſeparately. For as the action of ſuch forces ceaſes after the firſt impreſſion given; whatever direction and velocity theſe conjoint forces give the body, it will go on uniformly with that velocity, and in that direction ever after, by the firſt law of motion. Now at the beginning of the time, the body was ſuppoſed at *A*, at the end of the time it will be found in *D* by Cor. 1, therefore it will have deſcribed a right line paſſing through *A* and *D*, (that is the diagonal *AD*), with an uniform motion, and in the ſame time that by the former force or impact it would have deſcribed the ſide *AB* uniformly, or by the latter impact the ſide *AC*.

COR. III. Let both forces act continually, but uniformly, during the time in which the body by one force deſcribes *AB*, by the other force *AC*. In this caſe when the ſides are deſcribed ſeparately, they will each be deſcribed with a velocity uniformly accelerated; and becauſe the time in which theſe ſpaces are deſcribed is given, the accelerating forces will be as thoſe ſpaces, or the ſides *AB* and *AC*, by the firſt law of motion *. I ſay in this caſe, the body when acted upon by both forces conjointly, will deſcribe the Diagonal *AD*, with a velocity uniformly accelerated, and in the ſame time in which it would have deſcribed either ſide by its reſpective force ſeparately.

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* See Saunderſon on the deſcent of heavy bodies, Cor. iv. Canon firſt.

In the lines *AB* and *AC*, let *E* and *G* be two cotemporary places of the body when it deſcribes thoſe ſides ſeparately. Compleat the parallelogram *AEPG* and *P* will be the cotemporary place of the body when the forces act conjointly, by Cor. 1. But becauſe *B* and *C* are alſo cotemporary places of the body when the forces act ſeparately, we have by the firſt law of motion as before, *AE* : *AG* : : *AB* : *AC*. The parallelograms *AEPG* and *ABDC* are therefore ſimilar, and the point *P* lies in the diagonal *AD*. In like manner the place of the body at every inſtant, while the forces act conjointly will be found in the Diagonal. Moreover *AB* : *AD* : : *AE* : *AP*. If therefore the point *E* (repreſenting the place of the body) deſcribes the line *AB*, with a velocity uniformly accelerated, the point *P* will alſo deſcribe the diagonal *AD*, with the like kind of motion, and in the ſame time. Therefore the body when acted upon by both forces jointly will deſcribe the diagonal with a velocity uniformly accelerated, and in the ſame time in which it would have deſcribed either ſide by its reſpective force ſeparately.

SCHOLIUM. The motion of the body along the diagonal generated by the conjoint forces, is juſt ſuch as would have been produced by a ſingle force, acting continually and uniformly in the direction of the Diagonal *AD*, and whoſe quantity is to either of the other forces as the Diagonal is to the reſpective ſide. For the time being given, theſe accelerating forces, are as the ſpaces deſcribed, as was ſaid before.

COR. IV. Hence a third force acting in the contrary direction *DA*, and whoſe quantity is to either of the other two forces as the Diagonal, to the ſides reſpectively, would juſt counter-act the effect of the other two forces and keep the body in equilibrio.

COR. V. Hence if a body be acted upon by three forces, whoſe quantities and determinations are repreſented by the ſides of a Triangle *ABD*, that body will be at reſt.

SCHOLIUM. If the forces are one inſtantaneous and the other continual and uniform; the body at the end of the given time will be found in the oppoſite angle of the parallelogram as before, but the path it will have deſcribed will not be a right line, but a Parabola, as is ſhewn by all the writers on projectiles.

It may now be aſked how can ſuch a law as this be proved experimentally? By what ſort of trials are we to find out, whether the determination of two forces or the quantity of motion produced in ſuch determination, is, or is not altered by their acting in conjunction? It may be anſwered that all the immediate conſequences of this law that can be eſtabliſhed by experiment, are ſo many proofs of the principle from which they are derived. Nothing is more eaſy than to apply three weights ſo as to act at once upon one body; and that in any propoſed directions. Where-ever the quantity and directions of theſe forces, are ſo proportioned, as according to the corollary drawn from this law, to balance each other; there we ſhall find the body will be at reſt. All the experiments made on what are called the mechanical powers are alſo (by conſequence) proofs of this Law.

The firſt law of motion cannot be proved by direct Experiment. We never in fact do ſee any body whoſe motion always continues uniform and in a right line; but we juſtly infer from ſuch experiments as we can make, that it would do ſo were all impediments removed. Paradoxical as it may ſeem, yet it muſt be acknowledged, that thoſe circumſtances which it is alledged do take place in all motions, and therefore become *laws of motion*, are yet not to be ſeen (ſimply) in any one motion we are acquainted with. By making theſe laws ſo very general and abſtract, they exiſt in idea only; they do not admit of a direct proof from ſimple fact, and can only be inferred by way of conſequence from facts. An experiment cannot be contrived in which a body ſhall be placed in thoſe very ſimple circumſtances ſuppoſed in the law itſelf; foreign ones will attend it. But though ſome argument muſt be uſed, it is ſeldom that more than one ſtep is neceſſary, all beyond is matter of fact; ſo that theſe laws may be fairly ſaid, not to depend on metaphyſical reaſonings about matter and mo-

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tion, or uncertain hypotheſes; but as all ſound philoſophy ſhould do, on obſervation and Experiment.

Mac Laurin in his comment on this law, ſeems to repreſent it as affirming that when a force acts in part upon a body *A*, it produces only ſo much motion as the acting part, not the whole, can produce. There are many inſtances where a force which ſeemingly acts upon one body, really acts upon two. When the motion produced in the ſecond *B* is ſo ſlow as to be imperceptible, we then ſay part of the original force is ſuſtained by *B*, and part only acts upon *A*; but ſurely to ſay the motion generated in *A* is proportional to ſo much force as is impreſſed on *A*, or that "the acceleration is proportional to the power that acts," is identical. For we undoubtedly meaſure, the power that acts, by the acceleration it produces. It is identical conſidered as a law or rule.—It is quite another thing to enquire how we ſhall determine what proportion that part of the force which does act upon *A*, bears to the whole.

When Mac Laurin explains Newton's firſt corollary he *premiſes* this principle. "That when a body is acted upon by a motion are called the mechanical powers are alſo (by conſequence) proofs of this Law.

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When Mac Laurin explains Newton's firſt corollary he *premiſes* this principle. "That when a body is acted upon by a motion or power parallel to a right line given in poſition, this power or motion has no effect to cauſe the body to approach towards that right line or recede from it, but to move in a line parallel to that right line only; as appears from the ſecond law of motion." It is not eaſy to underſtand this, if the body be ſuppoſed to be acted upon by one force only. And if it relates to the caſe of a body acted upon by two forces at once, it will (when fully explained) probably coincide with the principle or Law before laid down. That this *principle* is or ſhould be the Law itſelf, we contend; but if this be denied, then we aſk, is this *principle* a lemma to be *premiſed* for demonſtrating the law, as is ſaid at firſt; or a *conſequence* from the law, as is ſaid at the concluſion? See Mac Laurin's account, p. 115 and p. 121. Quarto.

What Mr. *Milner* ſays in his excellent and judicious comment on the third Law of motion, may be fairly applied both to Mac Laurin, and even Newton himſelf, with reſpect to the ſecond law of motion. ——— "I cannot poſſibly conceive that ſo ſkilful and accurate a philoſopher could believe, that the *third* law of motion was an inference of reaſon, excluſive of all experiment; and yet, if words have any meaning at all, the above quotation inclines us to think ſo. A law of nature is not merely a deduction of reaſon: it muſt be proved, either at once and directly, by ſome ſimple and deciſive experiments; or if that cannot be done, by ſuch experiments as enable us to collect its exiſtence by the aſſiſtance of geometry." See Phil. Tranſ. Vol. 68. part 1. page 352.

P. S. Newton's ſixth definition is liable to the ſame ſort of objections. It is not quoted any where, nor does there ſeem any neceſſity for eſtabliſhing a meaſure of the abſolute quantity of centripetal force; but if there be; the *efficacy* of the cauſe that propagates it, cannot be applied for this purpoſe, as wanting a meaſure itſelf. Mr. *Thorpe* ſays "the abſolute quantity of centripetal force is meaſured by the motion generated in the ſame time in equal bodies, placed at equal diſtances from different central bodies;" (ſee his comment on Newton) but neither can this be admitted, unleſs the *Law* of thoſe centripetal forces be the ſame. If one centripetal force be in an inverſe duplicate, and the other in an inverſe triplicate ratio of the diſtances from the central bodies; then the proportion of their abſolute forces, meaſured in this manner, will vary, as thoſe *equal* diſtances vary: And there is one particular diſtance, *in every caſe*, in which the centripetal force in each ſyſtem will be equal; and therefore this is no rule whereby to determine the ratio of the abſolute centripetal forces in theſe two ſyſtems.

What has been ſaid is by no means intended to cavil at ſo great a man as Newton; but inaccuracies in the manner of reaſoning, create great difficulties to learners, and ſometimes to more experienced philoſophers. Of this we have a remarkable inſtance in the controverſy carried to a great length, between two able mathematicians, ROBINS, and JURIN, about Newton's *Proof* of his firſt Lemma to the laws of motion, owing to this; that Newton took that for a propoſition, and therefore to be *proved*, which is certainly nothing but a Definition, and therefore can only be *explained*.

THE END.

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